Kuwait University | Math 101 | December 10, 2000
Dept of Math & Comp Sci | Second Examination | Duration: 75 minutes

Answer the following questions. Calculators, Mobile Phones and Pagers are NOT allowed

(a) Let $f(x)=2x^2+x-2$ Use the definition of the derivative to find $f'(1)$. (3 points)

(b) Find the point(s) at which the tangent line to the graph of $3x^2-2y^2=1$ is perpendicular to the line $2x+3y+1=0$. (3 points)

2. (a) Find $f'(\theta)$ if $f(\theta)=\theta^2.\sec\theta-\dfrac{4}{\left(1+\sqrt[4]{\theta}\right)^3}$ (2 points)

(b) Let $y=r^2-6\sqrt{r+1}$, and $r=8\tan\dfrac{\pi}{t}$. Find $\dfrac{dy}{dt}$ at $t=4$. (2 points)

3. Let $f(x)=\begin{cases}\sin\pi x, & x<1,\\ A(x^2-1)+B, & x\geq 1.\end{cases}$ Find $A$ and $B$ so that $f'(1)$ exists. (3 points)

4. A right circular cylinder with closed top and bottom is being heated. Its height is increasing at a rate of 0.004 cm/min and its radius is increasing at a rate of 0.002 cm/min. At what rate is the total surface area changing when the cylinder has height 20 cm and radius 4 cm? (3 points)

5. Show that the equation $x^4+x^2+10x-9=0$, cannot have more than two real roots. (3 points)

6. Let $f(x)=3(2+x)\sqrt[3]{x}$ and note that $f'(x)=\dfrac{4x+2}{\sqrt[3]{x^2}}$ and $f''(x)=\dfrac{4x-4}{3x\sqrt[3]{x^2}}$.

Answer the following (1 point each)

(a) Show that the graph of $f$ has a vertical tangent.

(b) Determine the intervals on which $f$ is increasing, and the intervals on which $f$ is decreasing.

(c) What are the local extrema of $f$, if any?

(d) Determine the intervals on which the graph of $f$ is concave upward and the intervals on which the graph is concave downward.

(e) What are the points of inflection of the graph of $f$, if any?

(f) Sketch the graph of $f$ indicating vertical tangents, local extrema, concavity, points of inflection, and asymptotes, if any.